Introduction to ANOVA

- Up to now, we have used the mean to test the null hypothesis.
- We can use variance of the data for hypothesis testing
ANOVA

- F-test after Fisher
- Calculate F-obtained
- Compare to F-critical

**Sampling distribution of F**

- Gives all possible F values along with the p (F) for each value, assuming sampling is random from the population.

**Sampling distribution of F**

- Generated empirically by:
  - all possible samples of size $n_1$ and $n_2$
  - Estimating the population variance
  - Calculate $F_{obt}$ for all possible combinations of $s_1^2$ and $s_2^2$
  - Calculate p (F) for each different value of $F_{obt}$
ANOVA

- Use an ANOVA when you are comparing 2 groups or more.
- Use a one-way ANOVA when you have one independent variable with two or more levels.

ANOVA vs. Independent t-test

- T tests use means to examine differences
- ANOVA is an analysis of VARIANCE
- F can never be negative because all values are squared.
  - t can be positive or negative
  - F distribution is "folded" over t distribution
- \( t^2 = F \)
ANOVA vs. Independent t-test

- So, if you have one IV with 2 levels you can use:
  - Independent sample t test
  - Or ANOVA
- But if IV has more than 2 levels
  - Use ANOVA

Question

If you conduct an analysis of variance and find a negative $F$, it means that:
A) it is probably significantly different from 1
B) it is probably a Type I error
C) it is probably a Type II error
D) you have made a mistake in your calculations

Answer

If you conduct an analysis of variance and find a negative $F$, it means that:
A) it is probably significantly different from 1
B) it is probably a Type I error
C) it is probably a Type II error
D) you have made a mistake in your calculations
How ANOVA works

- An ANOVA tests the ratio of between-group variability to within-group variability

Sources of variability

<table>
<thead>
<tr>
<th>Between-Group Variance</th>
<th>Within-Group Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differences Between Group Means</td>
<td>Differences Between People's Scores Within each Group</td>
</tr>
<tr>
<td>Treatment Effect</td>
<td>Random Error</td>
</tr>
<tr>
<td>What we Can Explain</td>
<td>What we Can't Explain</td>
</tr>
</tbody>
</table>

The F-statistic

\[
F = \frac{\text{variability between groups}}{\text{variability within groups}}
\]

Or, in other words:

\[
F = \frac{\text{variance we can explain}}{\text{variance we can't explain}}
\]
Hypotheses for F-test

- The $F$-statistic is based on a one-tailed distribution.
  - Therefore, $F$-tests (ANOVAs) are always one-tailed
  - $H_0 = M_1 = M_2$
  - $H_1 = M_1 \neq M_2$

F table

- We use the $F$-table to find our critical value.
- Similar to $t$-tests, but now we have to look up two different degrees of freedom to find our critical value
- Found in back of book

Finding $F$ critical

- Find $df_{between}$ and $df_{within}$
  - $df_{between}$ is the df in the denominator
  - $df_{within}$ is the df in the numerator
  - Gives alpha .01 in first row
  - Alpha = .05 second row
Finding F critical

<table>
<thead>
<tr>
<th>df</th>
<th>df for your treatment effect</th>
<th>df for your error term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total df</td>
<td>a - 1</td>
<td>a(n - 1)</td>
</tr>
<tr>
<td>(an - 1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

n = total # of participants in each group
a = the number of groups

df between + df within = df total

Example
Therapy Experiment

A researcher wants to know what type of therapy helps people with a Simple Phobia the most. She assigns 15 people to 1 of 3 groups so that there are 5 people in each group: a psychoanalysis group, a cognitive therapy group, and a behavioral therapy group. After therapy, she has each person rate the severity of their phobia from 0 (it’s gone) to 10 (it’s the worst it has ever been).

<table>
<thead>
<tr>
<th>Psychoanalysis</th>
<th>Cognitive Therapy</th>
<th>Behavioral Therapy</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

X = 5  X = 4.4  X = 0.88
Example

Therapy Experiment

- Type of therapy is one IV with 3 levels:
  - psychoanalysis, cognitive therapy, and behavioral therapy
- Hypothesis
  - $H_0 = M_1 = M_2 = M_3$
  - $H_1 = M_1 \neq M_2 \neq M_3$

Performing an ANOVA

1. Find df
2. Calculate $F_{obs}$
3. Look up $F_{crit}$
4. Compare $F_{obs}$ to $F_{crit}$ to see if you have a significant effect

Calculating $F$ observed

- Remember that:
  - $F = \frac{\text{variability between groups}}{\text{variability within groups}}$
- So, to find $F$, we must calculate:
  - the between-group variability
  - the within-group variability
ANOVA summary table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
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</tr>
</tbody>
</table>

Calculating F observed

• In ANOVA, variance or variability is called the Mean Square (or MS for short)

• So $F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$

Calculating F observed

• $MS = \frac{SS}{df}$
  - SS is the Sum of Squares

• There are three SSs:
  - $SS_{\text{total}}$
  - $SS_{\text{between}}$ = (also called SS$_A$ or SS$_{\text{effect}}$)
  - $SS_{\text{within}}$ = (also called SS$_{\text{error}}$ or SS$_{\text{error}}$)
Getting our sum of squares

- Remember
- \( \Sigma(x - \bar{x})^2 \) = sum of squares
  - \( \frac{SS}{n} - 1 \) = variance
  - \( \sqrt{\text{variance}} \) = standard deviation

Calculating F obtained

- So, once we have our SS, we can divide by our degrees of freedom to get our MS between and MS error.
- Then we divide \( \frac{MS_{\text{between}}}{MS_{\text{error}}} \) to get our F-obtained.

Calculating ANOVA using the computational method
Comparing \( F \) obtained to \( F \) critical

• It's just like a one-tailed \( z \)-test or a one-tailed \( t \)-test!
• If \( F_{\text{obs}} > F_{\text{crit}} \), then you have a significant effect.
• If \( F_{\text{obs}} < F_{\text{crit}} \), then you do not have a significant effect.

Calculating ANOVA using Excel

• Demonstration of ANOVA using Excel

Interpreting a one-way ANOVA

• Type of therapy significantly affects symptoms of Simple Phobia.
  – Or, there is a \textit{main effect} of type of therapy on Simple Phobia.
Reporting the F-statistic

\[ F(\text{df between}, \text{df within}) = F_{\text{obs}}, \ p < .05 \]

Assumptions of ANOVA

1. The population from which the samples were taken are normally distributed
2. The samples are drawn from a population of equal variances (homogeneity of variance)

violations of Assumptions of ANOVA

- ANOVA, like t-test, is robust.
- Minimally effected by violations of normality
- Relatively insensitive to violations of homogeneity of variance
Effect size and ANOVA

- Remember that the effect size is a number that tells you how much variance in your dependent variable can be explained by your independent variable.
- How important is the effect?

Formula for effect size:

\[ \eta^2 = \frac{SS_{\text{between}}}{SS_{\text{total}}} \]

(\(\eta^2\) is pronounced "eta squared")

Power

- Power increases with larger sample sizes.
- Power varies directly with the size of the real effect.
  - The larger the real effect the larger the values of the \(SS\)
- Smaller variance = more power
- \(P = .05\)