Single sample t-tests

Introduction to single sample t-tests

- Z-tests were used when we had one sample and knew the population mean and standard deviation.
- With single sample t-tests, we have one sample and know the population mean but not the population standard deviation.


Sampling distribution of t

- Gives us:
  - All possible different t values for sample size of N
  - Probability of getting each value if sampling is random

T distribution

- Just like there is a z-distribution (or Standard Normal Curve), there is also a t-distribution...
- Entire family of t-distributions
  - One for every number of degrees of freedom

<table>
<thead>
<tr>
<th>z Test</th>
<th>t Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{calc} = \frac{\bar{X} - \mu}{\sigma/\sqrt{N}} )</td>
<td>( T_{calc} = \frac{\bar{X} - \mu}{s/\sqrt{N}} )</td>
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</tbody>
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where

\( s \) = estimate of \( \sigma \)  
\( s_\bar{X} \) = estimate of \( \sigma / \sqrt{N} \)
Degrees of freedom

- Degrees of freedom (df) are equal to (n-1).
- They refer to the number of values that are free to vary in calculating the statistic.

When to use a single sample t test

- When you don't know the population standard deviation (σ)
- Use a one-sample (or single-sample) t-test when you only have one sample
Formula for single sample t-test

\[ t = \frac{(X - \mu_{hyp})}{s_X} \]

- \( X \) = your sample mean
- \( \mu_{hyp} \) = your hypothesized population mean
- \( s_X \) = your estimated population standard error

Formula for single sample t-test

\[ s_X = \frac{s}{\sqrt{n}} \]

where

\[ s = \sqrt{\frac{\sum X^2 - (\sum X)^2}{n(n-1)}} \]
Example

The Psychomotor Development Index (PDI) was administered to a group of 56 low birth-weight (LBW) babies. Their mean on the PDI was 95.75 (SD = 12.58).

The population mean for this scale is 100. Is the psychomotor development of LBW babies significantly below the norm?

Example

• Null and alternative hypothesis
• One or two tailed test?
• # of degrees of freedom?
  – 56-1 = 55
• What is your t critical value?
  – t crit = 1.673 for alpha = .05

Example

• What is your t-obtained?
  – 95.75-100/ (12.58/√56) =-2.53
• Decision and conclusions
  – Reject the null, low birth weight babies scored lower than the norm.