

Queuing Theory Using QM

In this tutorial, we will cover the topic of queuing theory and will go over example problems of both single- and multiple-server queuing systems.

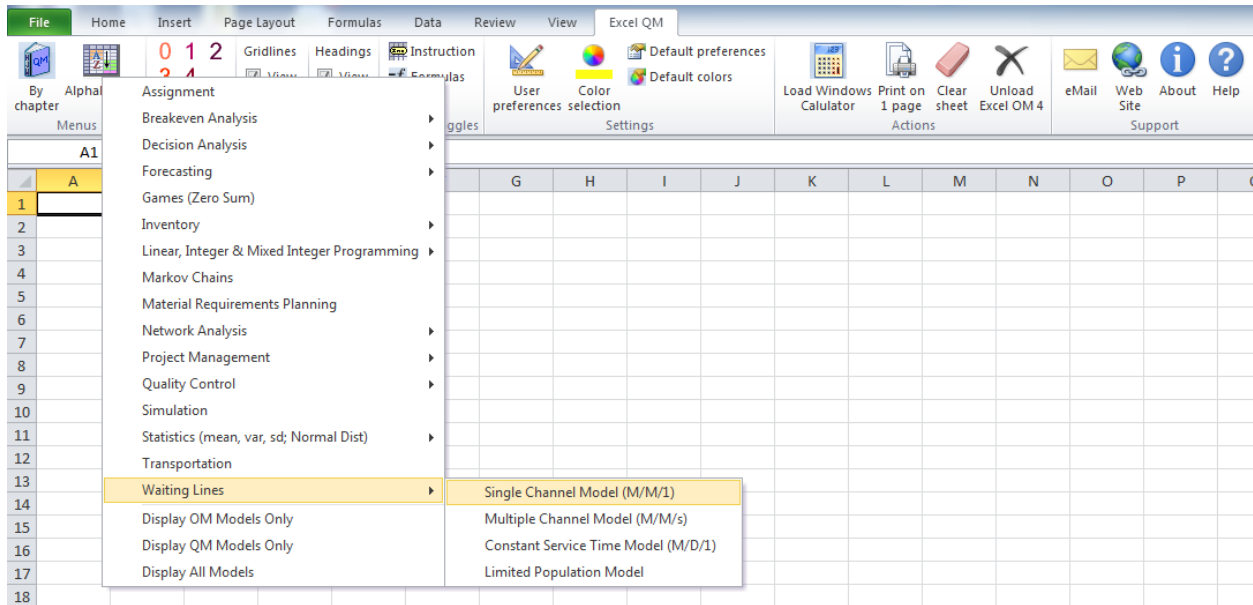
In our first problem, we will examine a single-server system. The Saint Leo University cheerleaders are holding a car wash as a fundraiser to buy new uniforms. The average time to wash a car is four minutes, and the time is exponentially distributed. Cars arrive at a rate of one every five minutes and the number of arrivals per time period is described by the Poisson distribution. Answer the following questions:

- A – What is the average time for cars waiting in the line?
- B – What is the average number of cars in the line?
- C – What is the average number of cars in the system?
- D – What is the average time in the system?
- E – What is the probability that there are more than three cars in the system?

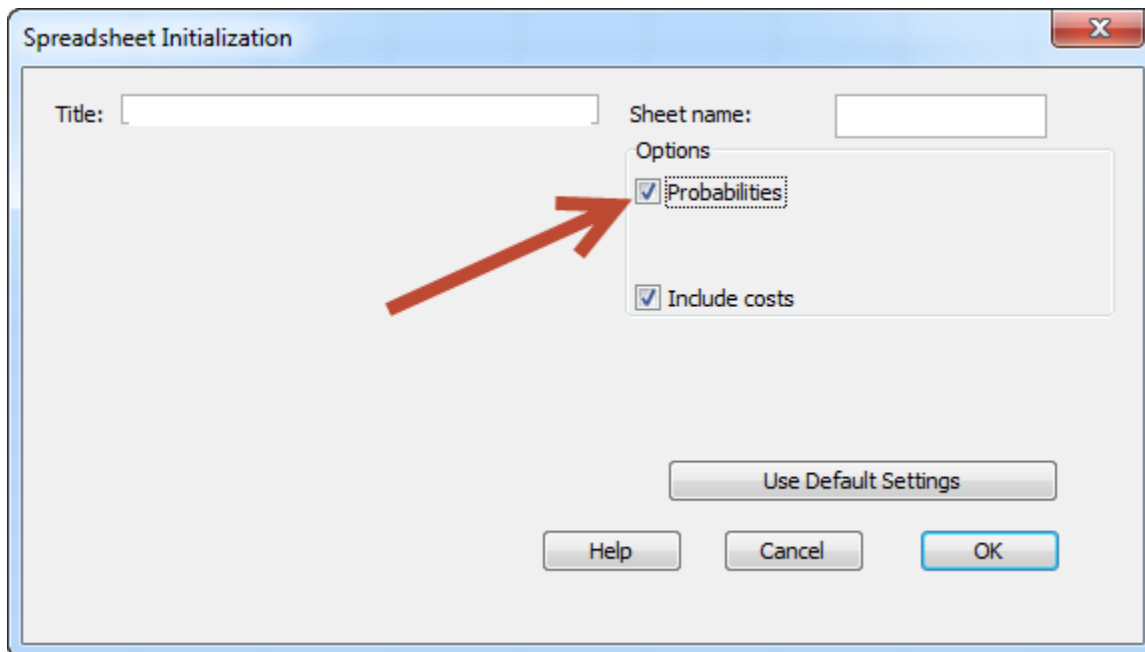
First, we need to examine our times and see if they are given in rates. *They must be given in units per measure of time, a rate.* If a car arrives every five minutes, we need to convert that over to an arrival rate – in this case if a car arrives every five minutes, we know that 12 will arrive in an hour ($60/5 = 12$). Now we have our arrival rate in terms of units per measure of time, or in this case, units per hour.

Similarly, we must examine our time to wash a car (our service time) and ensure it is reflected in the same manner. If we wash a car in four minutes, we know that we can wash 15 in an hour ($60/4 = 15$). Now we have our service rate in terms of units per measure of time, or in this case, units per hour.

Now we are ready to plug in our numbers to Excel QM. Open Excel QM, select the **Excel QM** tab → **Alphabetical** → **Waiting Lines** → **Single Channel Model**.



A Spreadsheet Initialization window will appear.



Be sure you tick the box for **Probabilities** and click **OK**.

A spreadsheet table will appear.

	A	B	C	D	E	F
1	Waiting Lines		M/M/1 (Single Server Model)			
2	The arrival RATE and service RATE both must be rates and use the same time unit. Given a					
3						
4	Data			Results		
5	Arrival rate (λ)			Average server utilization(ρ)	#DIV/0!	
6	Service rate (μ)			Average number of customers in the queue(L_q)	#DIV/0!	
7	Number of servers	1		Average number of customers in the system(L)	#DIV/0!	
8	Server cost \$/time			Average waiting time in the queue(W_q)	#DIV/0!	
9	Waiting cost (\$/time)			Average time in the system(W)	#DIV/0!	
10				Probability (% of time) system is empty (P_0)	#DIV/0!	
11	The service rate must be greater than the arrival rate			Cost - based on waiting	#DIV/0!	
12				Cost - based on system	#DIV/0!	
13						
14						
15	Probabilities					
16	Number	Probability	Cumulative Probability			
17	0	#DIV/0!	#DIV/0!			
18	1	#DIV/0!	#DIV/0!			
19	2	#DIV/0!	#DIV/0!			
20	3	#DIV/0!	#DIV/0!			
21	4	#DIV/0!	#DIV/0!			
22	5	#DIV/0!	#DIV/0!			
23	6	#DIV/0!	#DIV/0!			
24	7	#DIV/0!	#DIV/0!			
25	8	#DIV/0!	#DIV/0!			
26	9	#DIV/0!	#DIV/0!			
27	10	#DIV/0!	#DIV/0!			
28	11	#DIV/0!	#DIV/0!			
29	12	#DIV/0!	#DIV/0!			
30	13	#DIV/0!	#DIV/0!			
31	14	#DIV/0!	#DIV/0!			
32	15	#DIV/0!	#DIV/0!			
33	16	#DIV/0!	#DIV/0!			
34	17	#DIV/0!	#DIV/0!			
35	18	#DIV/0!	#DIV/0!			
36	19	#DIV/0!	#DIV/0!			
37	20	#DIV/0!	#DIV/0!			
38						
39						

Now we simply enter our arrival rate (12) and service rate (15) and the program will make the calculations for us (note that we are not given any costs here so we can just leave those fields blank):

Data

Arrival rate (λ)	12
Service rate (μ)	15
Number of servers	1
Server cost (\$/time)	
Waiting cost (\$/time)	

Results

Average server utilization(ρ)	0.8
Average number of customers in the queue(L_q)	3.2
Average number of customers in the system(L)	4
Average waiting time in the queue(W_q)	0.26667
Average time in the system(W)	0.33333
Probability (% of time) system is empty (P_0)	0.2
Cost - based on waiting	0
Cost - based on system	0

Probabilities

Number	Probability	Cumulative Probability
0	0.200000	0.200000
1	0.160000	0.360000
2	0.128000	0.488000
3	0.102400	0.590400
4	0.081920	0.672320

All we need to do now is look at the table above for our answers to the following questions:

- A. $W_q = 0.27$ hour
- B. $L_q = 3.2$
- C. $L = 4$
- D. $W = 0.33$ hour
- A. $P_{n>3} = 1$ minus the cumulative probability from table (less than or equal to three) = $1 - 0.5904 = 0.4096$

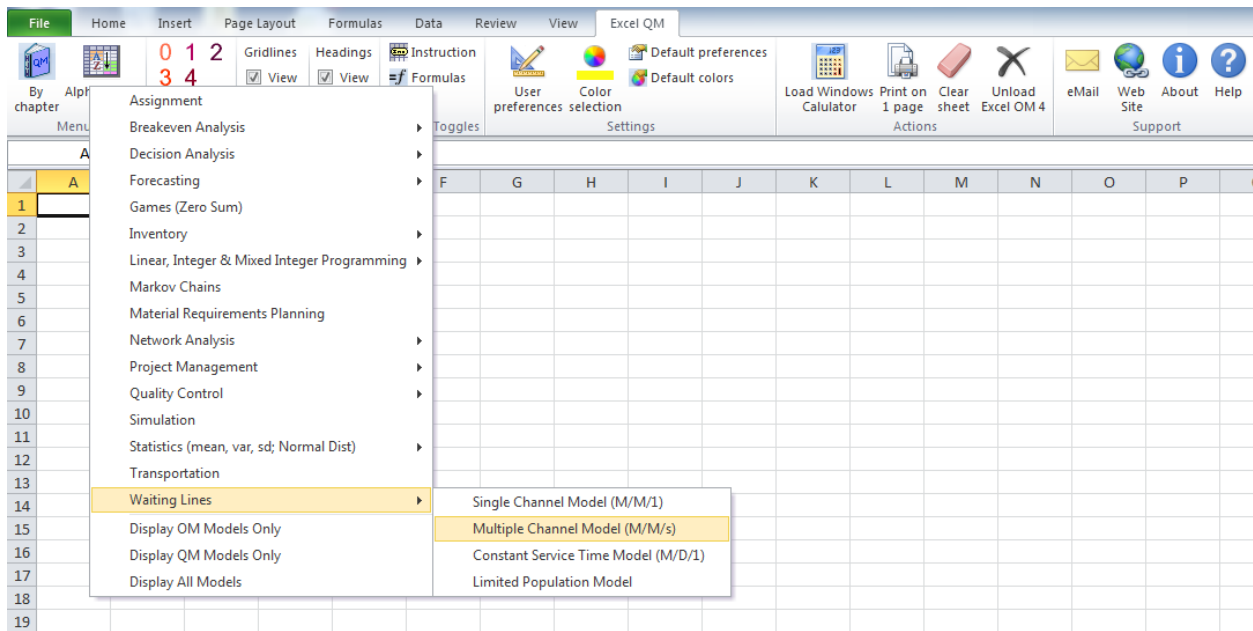
[Click here](#) to download the completed spreadsheet table so you can compare it to yours.

Next we will discuss multiple-server systems.

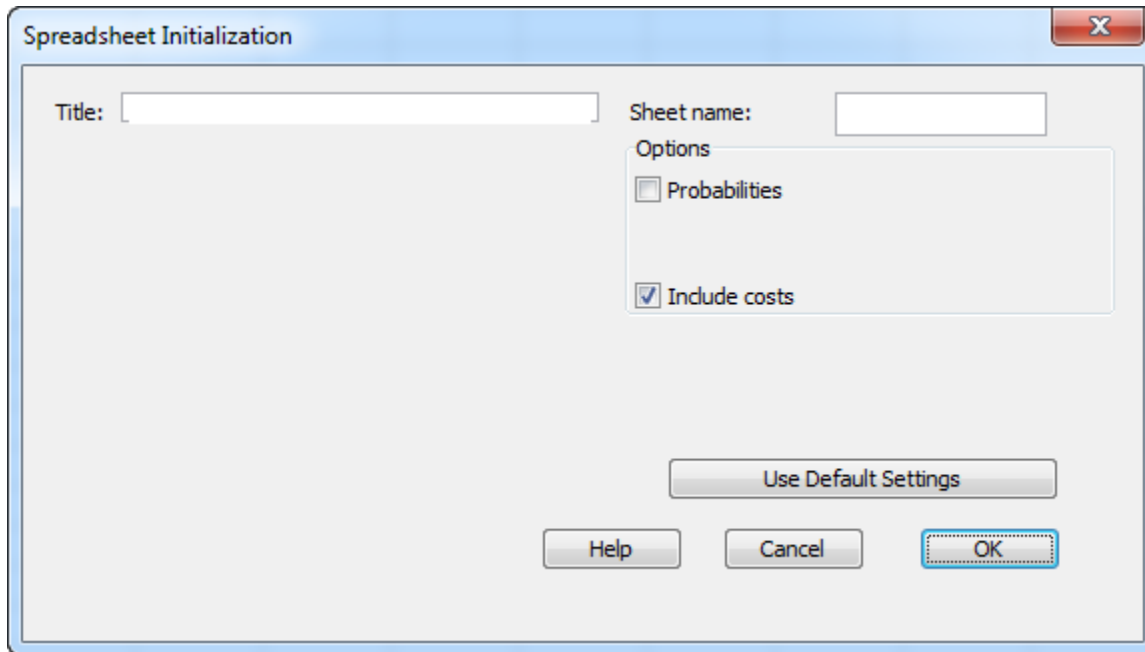
When additional band members arrived to help at the car wash, it was decided that two cars should be washed at a time instead of one. Both work crews would work at the same rate as described above. Answer the following questions:

- A – What is the average time for cars waiting in the line?
- B – What is the average number of cars in the line?
- C – What is the average number of cars in the system?
- D – What is the average time in the system?

Now we are ready to plug in our numbers to Excel QM. Open Excel QM, select the **Excel QM** tab → **Alphabetical** → **Waiting Lines** → **Multiple Channel Model**.



A Spreadsheet Initialization window will appear.



Click **OK**. A spreadsheet table will appear.

	A	B	C	D	E	F
1	Waiting Lines		M/M/s			
2	The arrival RATE and service RATE both must be rates and use the same time unit. Given a					
3						
4	Data			Results		
5	Arrival rate (λ)			Average server utilization(ρ)	#DIV/0!	
6	Service rate (μ)			Average number of customers in the queue(L_q)	#N/A	
7	Number of servers(s)			Average number of customers in the system(L)	#N/A	
8	Server cost \$/time)			Average waiting time in the queue(W_q)	#N/A	
9	Waiting cost (\$/time)			Average time in the system(W)	#N/A	
10				Probability (% of time) system is empty (P_0)	#N/A	
11	The total service rate (rate*servers) must be greater than the arrival rate			Cost - based on waiting	#N/A	
12				Cost - based on system	#N/A	
13						
14						
15						
16						
17						

Now we simply enter our arrival rate (12) and service rate (15) and we put in 2 for the number of servers. Again, the program will make the calculations for us (note that we are not given any costs here so we can just leave those fields blank):

Data

Arrival rate (λ)	12
Service rate (μ)	15
Number of servers(s)	2
Server cost (\$/time)	
Waiting cost (\$/time)	

Results

Average server utilization(ρ)	0.4
Average number of customers in the queue(L_q)	0.1524
Average number of customers in the system(L)	0.9524
Average waiting time in the queue(W_q)	0.0127
Average time in the system(W)	0.0794
Probability (% of time) system is empty (P_0)	0.4286
Cost - based on waiting	0
Cost - based on system	0

- A. $W_q = 0.013$ hour
- B. $L_q = 0.152$
- D. $L = 0.952$
- D. $W = 0.079$ hour

[Click here](#) to download the completed spreadsheet table so you can compare it to yours.

We can see that by adding a second group to washing the cars, our times drastically improved. Our average waiting time for cars in the line dropped from 0.27 to 0.013 hours or from around 15 minutes to less than a minute. We also went from > 3 cars in a line to < 1 , and went from 4 cars in the system at once to about 1 car. Lastly, we can see the average time in the system went from 0.33 to 0.079 hours, or from about 20 minutes to just under 5 minutes.

This concludes the tutorial on single- and multiple-server queuing systems using Excel QM.